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Exactly soluble model for a fractionalized Weyl semimetal

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I. INTRODUCTION

Electronic systems jointly described as topological fall into two conceptually distinct classes: short-range entangled, which can be transformed into a product state by means of local unitaries, and those characterized by long-range entanglement, which cannot. The former include symmetry-protected topological states (SPTs), including topological insulators or the Haldane phase [1], and Weyl and Dirac semimetals. The latter class is composed of topologically ordered [2] states characterized by topological ground-state degeneracies and fractionalization of quantum numbers; canonical examples thereof are fractional quantum Hall (FQH) states. While many SPTs originate in the properties of the noninteracting band structure and topological order crucially requires interactions, these subsets are not mutually exclusive: Symmetry-enriched topological phases such as a fractional topological insulator (FTI) [3–5] can arise due to the interplay of the two ingredients.

Recently, a great deal of attention, both in theory and in experiment, has been devoted to Weyl semimetals (see references in [6,7]). Their deceptive simplicity—a Weyl semimetal is essentially an accidental degeneracy in the band structure of a three-dimensional solid [8,9]—hides rich phenomenology: Fermi arcs [10], negative magnetoresistance [11,12], the quantized circular photogalvanic effect (CPGE) [13], etc. All of the above can be derived within the framework of noninteracting physics, but has been shown to be stable to interactions, which mostly move around and renormalize the Weyl cones [14–17]. On the other hand, interactions may generate topological order which cannot be described within the framework of simple band theory, possibly creating nontrivial fractionalized counterparts to gapped noninteracting topological states, as demonstrated by the FTI. Indeed, fractionalized phases that possess Weyl nodes have been postulated at the boundary of a four-dimensional quantum Hall insulator [18]. Reference [19] constructs a three-dimensional topologically ordered Weyl phase; however, fractionalization is not present in its low-energy theory.[1]

In this work, we provide a realization of a fractionalized Weyl semimetal in three dimensions by constructing an exactly solvable lattice Hamiltonian of a topologically ordered Weyl phase, whose low-energy theory features fractionally charged excitations. We show that the CPGE, a second-order response, is universally quantized to fractional values, in clear contrast to standard Weyl semimetal. We also show a violation of the Wiedemann-Franz law and a gap in the single-particle electronic spectral function, which provide experimental signatures.

II. SOLVABLE MODEL

Our construction is based on a flexible blueprint for an exactly solvable lattice model of a $\mathbb{Z}_m$ lattice gauge theory with conserved U(1) charge, which was previously introduced to study symmetry-enriched topological phases (in particular FTIs) and their properties in two and three dimensions

[1]Reference [15] also finds an "orthogonal" strongly interacting Weyl state within the slave-particle description, however its thermodynamic and transport properties are those of a standard Weyl semimetal. Reference [20] constructs a topologically ordered electric insulator with Majorana-Weyl cones of emergent fermions, with a (nonquantized) thermal Hall response, while Ref. [21] derives a fractionalized response for a semimetal based on a microscopic parton construction.

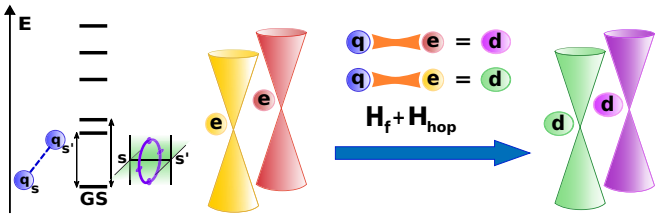

FIG. 1. Our system is made up of a topologically ordered bosonic model coupled to electrons. Prior to coupling, the spectrum of the bosonic model is discrete, and the low-energy theory is that of Weyl electrons e. The charging term in $\hat{H}_\mathrm{f}$ and the hopping $\hat{H}_\mathrm{hop}$ bind electrons to fractionally charged q_s bosons. Their composite d inherits the Weyl dispersion, while the original electrons become gapped (see Fig. 3).

[5,22,23]. It proceeds in two stages. First a bosonic model on a lattice is introduced, realizing a discrete lattice gauge theory, similar to a generalized toric code [24], albeit with the crucial distinction of having conserved boson number. The bosons carry a charge of $2e$ and can be thought of as paired electrons. This model features bosonic quasiparticles with fractional charge. In the second step, free electrons are added to the lattice and coupled to the bosonic quasiparticles. The band structure of the electrons encoded in the hopping amplitudes is chosen to be that of a Weyl semimetal; throughout this procedure the model remains exactly solvable. We show that the low-energy physics of the model can be mapped to a Weyl semimetal built out of fermions of fractional electric charge (see Fig. 1) and analyze the experimental consequences. Below we describe the model in more detail; further information about this class of constructions can be found in the Supplemental Material [25] and Refs. [5,22,23].

The bosonic degrees of freedom exist on the sites s and links $\langle ss'\rangle$ of a bipartite cubic lattice Λ; they carry a U(1) charge of $2e$. Creation and number operators on sites and links are denoted by $\hat{b}_s^\dagger$ and $\hat{n}_s$ and by $\hat{b}_{ss'}^\dagger$ and $\hat{n}_{ss'}$, respectively. The bosonic Hamiltonian comprises of two terms, which we refer to as the charging and the hopping Hamiltonian

$$\hat{H}_\mathrm{B} = V\sum_s \hat{Q}_s^2 - \frac{u}{2}\sum_p (\hat{B}_p + \hat{B}_p^\dagger), \tag{1}$$

where V and $u > 0$ are parameters of the model corresponding to the cluster charging and flux energy scales, respectively. The cluster charge operator $\hat{Q}_s$ measures the total charge on site s and the corresponding six links $\langle ss'\rangle$ of the three-dimensional lattice Λ (see Fig. 2),

$$\hat{Q}_s = \alpha_s \sum_{s'} \hat{n}_{ss'} + m\hat{n}_s, \tag{2}$$

with the constant $\alpha_s = 1\cdot\mathbb{1}_A(s) + (m-1)\cdot\mathbb{1}_B(s)$, where $\mathbb{1}_{A\,(B)}(s)$ is the indicator function for the A (B) sublattice. The integer $m \geqslant 2$ is a parameter determining the discrete gauge group $\mathbb{Z}_m$ and, consequently, values of all fractionalized quantities. Since $V > 0$, the charging term is a short-range repulsive interaction; $\hat{B}_p$, on the other hand, is a ring exchange term consisting of simultaneous hopping between adjacent sites and links of a plaquette,

$$\hat{B}_p = \hat{U}_{s_1s_2}\hat{U}_{s_2s_3}\hat{U}_{s_3s_4}\hat{U}_{s_4s_1}, \tag{3}$$

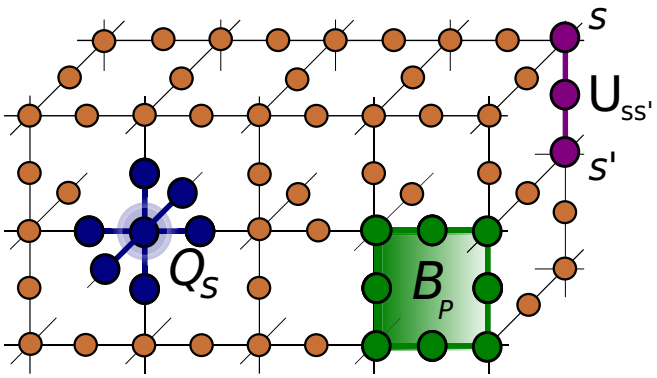

FIG. 2. Bosonic model. The bosons exist on a cubic lattice, both on the sites s and on the links $\langle ss'\rangle$. The cluster charge $\hat{Q}_s$ (2) measures the charge on the site s and the adjacent link $\langle ss'\rangle$; the ring exchange $\hat{B}_p$ (3) composed of four hopping operators $\hat{U}_{ss'}$ (4) acts on all sites and links of a plaquette p.

where the hopping term on a link is defined as

$$\hat{U}_{ss'} = (\hat{b}_s^\dagger)^{\alpha_s - 1}\hat{b}_{s'}^\dagger\hat{b}_{ss'}^{\alpha_s} + \hat{b}_{s'}^{\alpha_{s'}-1}\hat{b}_s(\hat{b}_{ss'}^\dagger)^{\alpha_{s'}}. \tag{4}$$

A crucial part of the model is the definitions of the link and site local Hilbert spaces. The $\hat{b}_{ss'}^\dagger$ degrees of freedom on the links are generalized hard-core bosons, whose occupation number $\hat{n}_{ss'}$ is restricted to lie in $\{0,\ldots,m-1\}$. The site bosons, in contrast, are in the rotor representation $\hat{b}_s^\dagger \sim e^{i\hat{\theta}_s}$, with $[\hat{n}_s, \hat{\theta}_s] = i$, and consequently their occupation is integer valued, $-\infty \leqslant n_s \leqslant \infty$.

With those definitions in place, the intuitive action of $\hat{B}_p$ is to change site occupations by $\pm 1 \bmod m$ on links around the plaquette while preserving the total boson number by compensating with hoppings between sites and links. The operator $\hat{U}_{ss'}$ hops a unit of cluster charge from one end of the link to the other:

$$[\hat{Q}_r, \hat{U}_{ss'}] = (\delta_{rs'} - \delta_{rs})\hat{U}_{ss'}. \tag{5}$$

Consequently, $\hat{Q}_s$ commutes with the product of $\hat{U}_{ss'}$ around any closed loop, which, in particular, implies

$$[\hat{Q}_s, \hat{B}_p] = 0\,\forall\, s, p. \tag{6}$$

Furthermore, since $\hat{U}_{ss'}^\dagger = \hat{U}_{s's} = \hat{U}_{ss'}^{-1}$ we have that $[\hat{B}_p, \hat{B}_{p'}] = [\hat{B}_p, \hat{B}_{p'}^\dagger] = 0$. Since also $[\hat{Q}_s, \hat{Q}_{s'}] = 0$, all terms in the Hamiltonian (2) commute and thus can be simultaneously diagonalized. The spectra of both operators are discrete: $\hat{Q}_s$ trivially, since it counts the number of bosons, and the hopping term $\hat{B}_p$ due to the relation $\hat{B}_p^m = 1$. The eigenvalues of $\hat{B}_p$ are consequently given by $b_p = e^{2\pi i/m}$. The quantum numbers $q_s \in \mathbb{Z}_{\geqslant 0}$ and $b_p \in \mathbb{Z}_m$ satisfy the constraints of a global $\sum_s q_s \equiv 0 (\mathrm{mod} m)$ and a set of local constraints $\prod_{p\in\mathcal{C}} b_p = 1$ for every cube $\mathcal{C}$ of the lattice. Labeling the eigenstates by $|\{q_s, b_p\}\rangle$, we obtain the energy spectrum

$$E_{\{q_s,b_p\}} = V\sum_s q_s^2 - \frac{u}{2}\sum_p (b_p + b_p^*). \tag{7}$$

This bosonic spectrum is discrete even in the thermodynamic limit and hence gapped, with the ground state $|\text{g.s.}\rangle = |\{q_s = 0, b_p = 1\}\rangle$.

There are two types of excitations. The first is charge excitations, created by string operators defined as $\hat{W}(\mathcal{L}) := \prod_{ss'\in\mathcal{L}} \hat{U}_{ss'}$, where $\mathcal{L}$ is any open path/string on the lattice Λ. Acting on the ground state gives rise to a pair of excitations localized on the two ends of the string [as seen from Eq. (5)], each costing an energy V. The second kind is $\mathbb{Z}_m$-flux loops on the dual lattice Λ^*.

The topological content of the model (1) is that of the $\mathbb{Z}_m$ gauge theory [24], albeit in an unconventional charge-conserving form which will prove suitable for coupling to a fermionic sector. Another key property is the fact that the bosonic charge excitation is also fractionally charged under U(1) with a charge of $2e/m$ (see [5,25]).

To study the interplay of a band-theoretic fermion system with a topological order we couple the bosons in (1) to additional fermionic degrees of freedom on the sites of Λ. The total Hilbert space is spanned by the states in the tensor product of the electronic occupation and the bosonic Hilbert space, i.e., $|\{n\}\rangle_\text{f} = |\{n_{s\sigma}\}\rangle_\text{e} \otimes |\{n_s, n_{ss'}\}\rangle$. We denote by $\hat{c}^\dagger_{s\sigma}$ and $\hat{n}_{s\sigma}$ the creation and number operators for the electrons of spin σ, and by $\hat{n}_{s,e}$ the total number of electrons on site s. The extended Hamiltonian takes the form

$$\hat{H}_\text{f} = V \sum_s \hat{\tilde{Q}}_s^2 - \frac{u}{2} \sum_p (\hat{B}_p + \hat{B}_p^\dagger) - \mu \sum_{s\sigma} \hat{n}_{s\sigma}, \tag{8}$$

with μ the chemical potential and $\hat{\tilde{Q}}_s = \hat{Q}_s - \hat{n}_{s,e}$.

Since all the commutation relations hold with $\hat{Q}_s$ replaced by $\hat{\tilde{Q}}_s$, the model is diagonalized like before in terms of the eigenstates $|\{\tilde{q}_s, b_p, n_{s\sigma}\}\rangle$ with $n_{s\sigma} \in \{0, 1\}$. The ground state of the fermionic model is given by $|\text{g.s.}\rangle_\text{f} = |\{\tilde{q}_s = 0, b_p = 1, n_{s\sigma} = 0\}\rangle$ when $\mu < 0$. In addition to the charge and flux excitations we also have the spin-$\frac{1}{2}$ fermionic excitation, where $n_{s\sigma} = 1$ for some site s and spin $\{\sigma = \uparrow, \downarrow\}$. As previously, the electrically charged particles satisfy the global constraint $\sum_s \tilde{q}_s + \sum_s n_{s,e} = 0 (\text{mod } m)$. It is easy to see that in the ground state each electron is accompanied by a bosonic charge to yield together $\tilde{q}_s = 0$. Furthermore, it is possible to write a nearest-neighbor hopping Hamiltonian which commutes with $\hat{H}_\text{f}$ and moves them together without dissociating,

$$\hat{H}_\text{hop} = -\sum_{\langle ss'\rangle} \sum_{\sigma\sigma'} [t_{ss',\sigma\sigma'} \hat{c}^\dagger_{s'\sigma'} \hat{c}_{s\sigma} \hat{U}_{ss'} + \text{h.c.}], \tag{9}$$

where the hopping amplitudes $t_{ss',\sigma\sigma'}$ define the band structure. The boson-electron composites are thus stable and the low-energy excitations are fermions of fractional charge $q_\text{f} = e(1 + 2/m)$. In what follows we choose

$$\begin{aligned} t_{s,s+\hat{\boldsymbol{e}}_\delta} &= i\sigma_\delta - \sigma_z, \quad \delta = x, y, \\ t_{s,s+\hat{\boldsymbol{e}}_z} &= -\sigma_z, \quad t_{s,s} = M\sigma_z, \end{aligned} \tag{10}$$

with all other $t_{s,s} = 0$ if they are not related by Hermiticity to the ones given. In momentum space, this model has a Weyl semimetal band structure if the real parameter $M \in (-3, 3)$. The Bloch Hamiltonian reads $\mathcal{H}(\boldsymbol{k}) = \sin(k_x)\sigma_x + \sin(k_y)\sigma_y + [M - \cos(k_x) - \cos(k_y) - \cos(k_z)]\sigma_z$, where $\sigma_{x,y,z}$ are the Pauli matrices acting on spin degrees of freedom.

III. FRACTIONAL CURRENTS

Charge fractionalization is manifest in the currents. The charge operator on site s [whose sum over all sites, via Eq. (2), is the total electric charge of elementary bosons and electrons in the system] is defined as $\hat{\rho}_s = \frac{2}{m}\hat{Q}_s + \sum_\sigma \hat{c}^\dagger_{s\sigma}\hat{c}_{s\sigma}$,

$$\hat{\rho}_s = \frac{2}{m}\hat{Q}_s + \sum_\sigma \hat{c}^\dagger_{s\sigma}\hat{c}_{s\sigma}, \tag{11}$$

and from the continuity equation for this local quantity,

$$\frac{d}{dt}\hat{\rho}_s = \frac{i}{\hbar}[\hat{H}, \hat{\rho}_s] = -\sum_{\delta=x,y,z} \left(\hat{J}^\delta_{s+\hat{\boldsymbol{e}}_\delta} - \hat{J}^\delta_s\right), \tag{12}$$

we obtain that the charge current is given by

$$\hat{J}^\delta_s = \frac{i}{\hbar}\frac{m+2}{m} \sum_{\sigma\sigma'} (t_{(s-\hat{\boldsymbol{e}}_\delta)s,\sigma\sigma'} \hat{c}^\dagger_{s\sigma} \hat{c}_{(s-\hat{\boldsymbol{e}}_\delta)\sigma'} \hat{U}_{(s-\hat{\boldsymbol{e}}_\delta)s} - \text{h.c.}).$$

The total currents $\hat{J}^\delta$ are obtained by summing over all sites s. Since $\hat{J}^\delta$ commutes with $\hat{\tilde{Q}}_s$ and $\hat{B}_p$, its matrix elements in the basis $|\{\tilde{q}_s, b_p, n_{s\sigma}\}\rangle$ are diagonal in the $\tilde{q}_s$ and b_p sectors. We therefore restrict the study to the low-energy sector $\tilde{q}_s = 0$ and $b_p = 1$, where the only excitations are fermionic composites of bosonic quasiparticles and electrons. A state in this sector, given by an arbitrary electronic configuration, can be written as $|\{n_{s\sigma}\}\rangle_\text{f} = |\{\tilde{q}_s = 0, b_p = 1, n_{s\sigma}\}\rangle$. It is easily shown (see [25]) that $\hat{c}^\dagger_{s\sigma}\hat{c}_{(s-\hat{\boldsymbol{e}}_\delta)\sigma'}\hat{U}_{(s-\hat{\boldsymbol{e}}_\delta)s} |\{n_{r\xi}\}\rangle_\text{f} = \pm |\{n'_{r\xi}\}\rangle_\text{f}$, where $n'_{r\xi} = n_{r\xi} + \delta_{rs}\delta_{\sigma\xi} - \delta_{r(s-\hat{\boldsymbol{e}}_\delta)}\delta_{\sigma'\xi}$, and the sign depends on the ordering of creation operators in the definition of the electronic basis state. Thus, in the low-energy subspace the matrix elements of the hopping/current operator are those of free fermions and we can define

$$\hat{d}^\dagger_{s\sigma}\hat{d}_{s'\sigma'} \equiv \hat{c}^\dagger_{s\sigma}\hat{c}_{s'\sigma'}\hat{U}_{s's}, \tag{13}$$

where $\hat{d}^\dagger_{s\sigma}$ creates a fractionally charged fermionic excitation (which braids nontrivially with the looplike $\mathbb{Z}_m$ flux excitations when not restricted to low-energy subspace). With this definition, the effective quasiparticle current in the low-energy subspace is given by

$$\hat{J}^\delta = \frac{m+2}{m}\frac{i}{\hbar} \sum_{s\sigma\sigma'} (t_{(s-\hat{\boldsymbol{e}}_\delta)s,\sigma\sigma'} \hat{d}^\dagger_{s\sigma} \hat{d}_{(s-\hat{\boldsymbol{e}}_\delta)\sigma'} - \text{h.c.}). \tag{14}$$

Having derived the low-energy effective theory, we can consider physical signatures. A standard calculation using the Kubo formula and the current (14) in momentum space yields the conductivity

$$\sigma_{xy} = \left(\frac{m+2}{m}\right)^2 \frac{e^2}{\pi h}\Delta k \equiv \frac{q_\text{f}^2}{\pi h}\Delta k. \tag{15}$$

While having a fractional prefactor, σ_{xy} also depends on the distance between the Weyl nodes in the z direction Δk. It is therefore not universal, and hence cannot serve as an unambiguous signature of the state. Crucially though, there exists a fractional quantized response which *is* universal.

IV. FRACTIONAL CPGE

The part of a photocurrent whose direction switches depending on the circular polarization of the incident light is called the circular photogalvanic effect [13,26]. It is given by the second-order response

$$\frac{dj_m}{dt} = \beta_{mn}(\omega)[\boldsymbol{E}(\omega) \times \boldsymbol{E}^*(\omega)]_n, \tag{16}$$

with $\boldsymbol{E}(\omega) = \boldsymbol{E}^*(-\omega)$ the electrical field. In an unfractionalized Weyl semimetal, the CPGE current is quantized in terms of universal constants e, h, c, and ε_0 and the monopole charge of the Weyl nodes, provided they lie at different energies (the assumption holds in enantiomeric crystals, with the inversion and all mirror symmetries broken) [13]. We show that the CPGE is quantized to a fractional value in our model, and hence provides a sharp diagnostic of the state.

We note that within the solvable model [5] coupling to the electromagnetic (EM) field is incorporated by introducing vector potentials $A^1_{ss'}$ and $A^2_{ss'}$ on the two halves of the links of the lattice and modifying the hopping operators, in particular $U_{ss'}$, via the Peierls substitution (with phases dependent on charges of elementary bosons/electrons being hopped). For weak and possibly time-dependent fields, the resulting low-energy effective theory is shown to be

$$\begin{aligned}\hat{H}_{\text{eff}} = &-\sum_{\langle ss'\rangle}(t_{ss',\sigma\sigma'}\hat{d}^\dagger_{s'\sigma'}\hat{d}_{s\sigma}e^{ie(1+2/m)A_{ss'}} + \text{h.c.}) \\ &- \mu\sum_{s\sigma}\hat{d}^\dagger_{s\sigma}\hat{d}_{s\sigma},\end{aligned} \tag{17}$$

i.e., the fermions $\hat{d}^\dagger_{s\sigma}$ minimally couple to the EM field, with coupling strength given by their fractional charge. Consequently, the tensor $\beta_{mn}(\omega)$ is derived as in Ref. [26], with the elementary electric charge replaced by the fractional q_{f}. More concretely, in a two-band model [13]

$$\beta_{mn}(\omega) = \frac{\pi q_{\text{f}}^3}{\hbar^2 V}\sum_{\mathbf{k}}(\partial_{k_m}\Delta E_{\mathbf{k}})\Omega_{\mathbf{k},n}\delta(\hbar\omega + \Delta E_{\boldsymbol{k}}), \tag{18}$$

where V is the sample volume, $\Delta E_{\boldsymbol{k}}$ is the energy difference between the valence and conduction bands, and $\Omega_{\boldsymbol{k}}$ is the Berry curvature of the valence band.

The δ function in Eq. (18) selects a k-space surface $\mathcal{S}$, and the trace of the CPGE tensor measures the Berry flux through that surface [13]. If, therefore, $\mathcal{S}$ encloses a Weyl node, Eq. (19) yields the monopole charge of that node, and hence CPGE is quantized *and* fractionalized,

$$\text{Tr}[\beta(\omega)] = i\frac{q_{\text{f}}^3}{2h^2}\oint_{\mathcal{S}} d\boldsymbol{S}\cdot\boldsymbol{\Omega} = i\pi\frac{q_{\text{f}}^3}{h^2}C, \tag{19}$$

where C denotes the monopole charge of the Weyl node.

V. WIEDEMANN-FRANZ LAW VIOLATION

Another signature of fractionalization of the low-energy degrees of freedom is the violation of the Wiedemann-Franz law, which may be anticipated from Eq. (14) (see [27]). Indeed, a short computation of thermal and electrical conductivities based on semiclassical Boltzmann approach, in the spirit of Ref. [28], yields

$$\frac{\sigma_{xx}}{\kappa_{xx}} = T\frac{\pi^2}{3}\left(\frac{k_{\text{B}}}{q_{\text{f}}}\right)^2, \tag{20}$$

where the fractional q_{f} takes the place of the electron charge. This is akin to what happens to the Hall conductivity in a FQH state. We stress, though, that our result is valid at zero magnetic field.

VI. ELECTRONIC GREEN'S FUNCTION

The low-energy physics is that of Weyl composites. What becomes of the electrons? Intuitively, it is clear that they can be created by dissociating the composite, at a finite energy cost. This is indeed reflected in a gap in the electron spectral function $A(r, r', \omega) = 1/\pi\ \text{Im}G^{\text{R}}(r, r', \omega)$, where the retarded electronic Green's function

$$G^R(r, t; r', t') = -i\theta(t - t')\langle\{\hat{c}_{r\sigma}(t), \hat{c}^\dagger_{r'\sigma'}(t')\}\rangle \tag{21}$$

is evaluated in the ground state of $\hat{H}_{\text{f}} + \hat{H}_{\text{hop}}$, and $\hat{c}_{r\sigma}(t) = e^{i(\hat{H}_{\text{f}}+\hat{H}_{\text{hop}})t}\hat{c}_{r\sigma}e^{-i(\hat{H}_{\text{f}}+\hat{H}_{\text{hop}})t}$. Evaluation of $\langle\text{g.s.}|\hat{c}^\dagger_{r\sigma}e^{-i(\hat{H}_{\text{f}}+\hat{H}_{\text{hop}})(t-t')}\hat{c}_{r'\sigma'}|\text{g.s.}\rangle$ is a nontrivial step. Nevertheless, it can be computed explicitly: The key insight is that $|\text{g.s.}\rangle$ is a superposition, generated by $\hat{H}_{\text{hop}}$, of electron-boson composites $\hat{d}$ and that $\hat{c}_{r\sigma}|\text{g.s.}\rangle$ is likewise a superposition of composites with an unpaired boson q_r at site r. Furthermore, $e^{-i\hat{H}_{\text{hop}}t}$ can only move a boson if accompanied by an electron; hence the matrix element vanishes unless $r = r'$ and then it reduces to the free fermion calculation, with electrons replaced by composites $\hat{d}$. Proceeding in the standard fashion, we obtain

$$A_{\sigma\sigma'}(r, r', \omega) = -\frac{1}{\pi}\text{Im}\left[\delta_{rr'}\delta_{\sigma\sigma'}\sum_{\boldsymbol{k}}\frac{u^*_\sigma(\boldsymbol{k})u_\sigma(\boldsymbol{k})}{\omega - \xi_{r,\sigma}(\boldsymbol{k}) + i\eta}\right],$$

where $u_\sigma(\boldsymbol{k})$ is the Bloch eigenstate of the valence band of Eq. (10). Crucially, $A(r, r', \omega) \propto \delta_{rr'}$, which trivializes its momentum dependence (see Fig. 3). Moreover, the effective dispersion $\xi_{r,\sigma}(\boldsymbol{k})$ is shifted with respect to the free electron dispersion of the valence band $\varepsilon(\boldsymbol{k}) - \mu$:

$$\xi_{r,\sigma}(\boldsymbol{k}) = V(1 - 2q_r + 2n_{r,\sigma}) + \varepsilon(\boldsymbol{k}) - \mu. \tag{22}$$

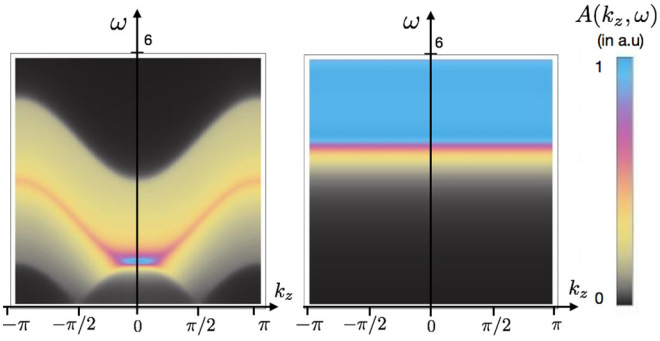

FIG. 3. Electron spectral functions numerically summed over k_x and k_y for a noninteracting Weyl semimetal (left panel) described by (10) and a fractional Weyl semimetal (right panel), plotted as a function of ω and k_z. In the numerical sum, we use the parameters $M = 2$, $\eta = 0.2$, $\mu = 0$, and $V = 3$.

This results in a spectral gap in the spectral function (see Fig. 3), which is a strong experimental signature. Interestingly, it also suggests an enhanced stability of the CPGE to hopping disorder (compared to the noninteracting case).

VII. CONCLUSION

We introduced a solvable lattice model for a fractional Weyl semimetal and derived experimental signatures of this exotic phase of matter. Our construction shows that such a phase combining fractionalization with a gapless Weyl band structure can be realized in strongly interacting three-dimensional systems and allows us to derive distinctive experimental signatures of this phase: gapped electronic quasiparticles, but a fractional CPGE and a fractionally violated Wiedemann-Franz law heralding the gapless nature of the fractional Weyl semimetal.

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